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VERIFICATION OF GENERALIZED FINITE DIFFERENCE METHOD FOR STATIC ANALYSIS OF CIRCULAR PLATES

Dilnaz Ye. Yerbolatova[1*], Irina M. Polyakova[1]

[1]International Educational Corporation, 050043, Almaty, Kazakhstan

*Corresponding author, e-mail: eerbolatovad@gmail.com

ABSTRACT

Introduction. Circular plates are structural elements widely used in construction and mechanical engineering. Accurate prediction of their deflections and bending moments under various loading and boundary conditions is essential for safe and economical design. **Methods and Materials.** This study applies the generalized finite difference method (GFDM), developed by R.F. Gabbasov, to the static calculation of thin isotropic circular plates in polar coordinates. The method extends the classical finite difference approach by introducing correction terms that account for discontinuities in loads and derivatives, enabling accurate treatment of concentrated forces and ring loads without artificial mesh refinement. The governing system is formulated as two coupled second-order Poisson equations for bending moments and deflections. Numerical implementation is performed in Mathcad and MS Excel. **Results and Discussion.** An illustrative calculation of a simply supported reinforced concrete circular plate under ring loading is presented in detail, demonstrating the step-by-step assembly of the finite difference equations into a matrix system. The GFDM yields maximum radial moment M_R = 967.8 kg·cm/cm' (analytical: 937.5, error 3.13%) and maximum circumferential moment M_θ = 2317.4 kg·cm/cm' (analytical: 2062.5, error 10.99%). Convergence analysis for 18 loading schemes confirms that GFDM achieves errors below 5% for smooth loads and below 15% for discontinuous loads upon mesh refinement. **Conclusions.** The GFDM is a computationally efficient and sufficiently accurate method for engineering calculations of circular plates under static loads, particularly advantageous for problems with discontinuous or locally applied loads.

KEYWORDS: circular plate; finite difference method; polar coordinates; bending moment; static analysis; convergence.

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1 INTRODUCTION

Circular plates are two-dimensional structural elements whose thickness is significantly smaller than their radius. They are encountered in a wide range of engineering applications: foundation slabs, pressure vessel covers and diaphragms, rotating discs in machinery, acoustic membranes, and roof elements [1]. The structural efficiency of plate action combined with the geometric simplicity of the circular form makes circular plates both practically important and analytically tractable. The classical theory of thin plates provides analytical solutions for circular plates under axisymmetric loading and standard boundary conditions - clamped, simply supported, or free edge [2, 3]. However, these solutions are restricted to simple geometry and idealized loading. In real engineering practice, plates frequently exhibit combined boundary conditions, locally distributed or ring loads, and concentrated forces, for which closed-form solutions are unavailable or excessi vely complicated [4]. The finite element method (FEM) is the most general numerical approach [5, 6], but for thin circular plates it requires careful selection of element types and mesh density to avoid shear locking[7]. The finite difference method (FDM) offers a transparent, computationally in expensive alternative for regular domains [8, 9], but the classical FDM faces difficulties with discontinuous loads. Gabbasov [10] proposed a generalized finite difference method (GFDM) that over comes this limitation by incorporating correction terms representing finite jumps of the function, its first derivative, and the distributed load across element boundaries. This approach was subsequently applied by Ibrahim (Mansour) [11] to circular plate problems, demonstrating accuracy and computational efficiency. The present work applies the GFDM to a representative set of static loading and boundary condition schemes for thin isotropic circular plates. Results are compared with analytical solutionsand SOFiSTiK FEM calculations, and practical guidelines are formulated

2 METHODS AND MATERIALS

The governing equations for bending of a thin isotropic circular plate of constant stiffness,

$$D=\frac{E\delta^3}{[12(1-\nu^2)]} \quad (1)$$

are formulated in polar coordinates (r, θ). For static problems, the GFDM is applied through a two-equation approach following Varvak [12]. Taking the bending moment m as the primary unknown, the system consists of two second-order Poisson equations:

$$\nabla^2 M = -p \quad (2)$$

$$\nabla^2 w = \frac{-M}{D} \quad (3)$$

where p is the distributed load and w is the deflection. This avoids the fourth-orderbiharmonic stencil and reduces computational complexity. The generalized finite difference equation for the Laplace operator at node (i, k) in a polar grid with radial step h and angular step θ is derived by averaging contributions from the four surrounding elements and introducing correction terms that represent jump discontinuities at element boundaries [10]. For the axisymmetric case, the stencil reduces to a three-point radial formula supplemented by a correction for the jump in the radial derivative. The central node (r = 0) is treated separately using the mean-value theorem [12]. Boundary conditions for the simply supported edge (w = 0, m = 0) are imposed using afictitious node with w' = -w. For the clamped edge (w = 0, $\partial w/\partial r = 0$), the fictitious node is assigned w' = w. Static calculations were implemented in Mathcad and MS Excel. FEM reference calculations were carried out in SOFiSTiK. Three grid densities were considered: h = l/8, h = l/16, and h = l/24.

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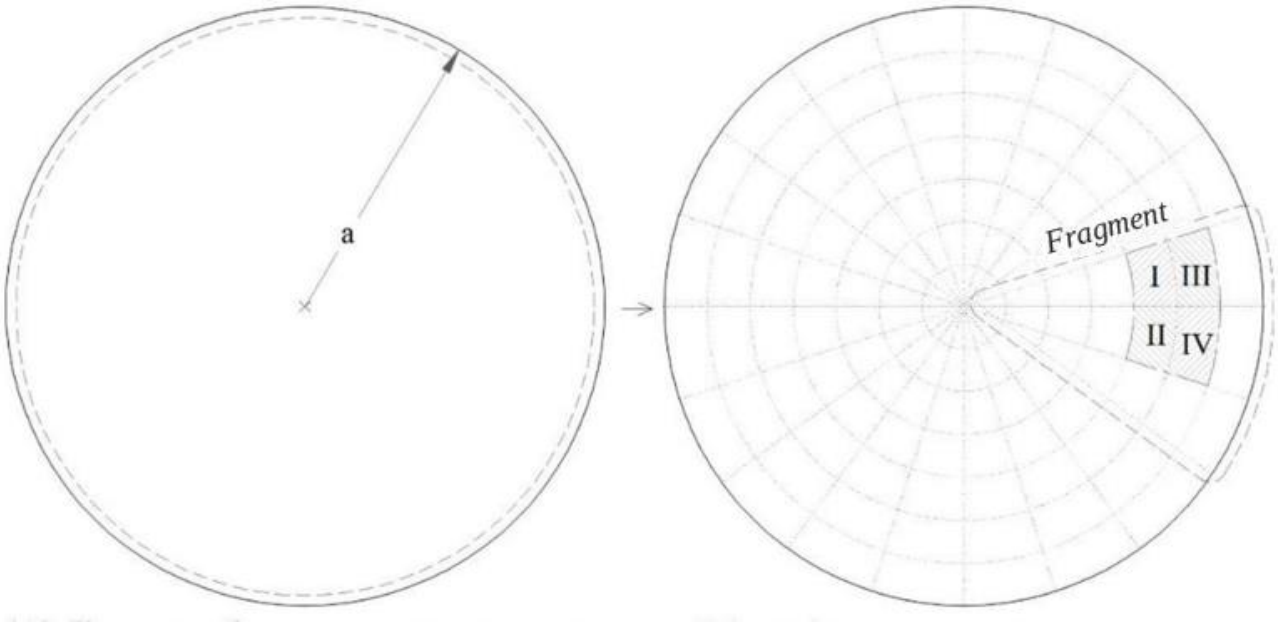


Fig. 1. Circular integration domain divided by a uniform polar grid

3 RESULTS AND DISCUSSION

Illustrative Example

A simply supported reinforced concrete circular plate ($E = 1.9\times10^5$ kg/cm², $\nu = 1/6$, $\delta = 12$cm, $a = 140$ cm) subjected to a uniformly distributed ring load of 3000 kg·cm/cm' applied at themid-radius is considered. The plate stiffness is,

$$D=\frac{190000\cdot(12)^3}{[12\cdot(1-1/36)} \approx 28141714 \text{ kg} \cdot \text{cm}.$$

The radial grid with

$$h = \frac{a}{6} = 23⅓ \text{ cm}$$

and six angular sectors yields seven radial calculation points (0 through 6, where point 6 is the boundary). For each interior point, two equations are assembled: one for the moment unknowns M_0 through M_6 from $\nabla^2 m = -p$, and one for the deflection unknowns W_0 through W_5 from $\nabla^2 w = m/D$. For the central point (0), the mean-value theorem yields $M_1 - M_0 = 0$ and

$$\frac{(h^2\cdot M_0)}{(4D)} = W_1 - W_0 \qquad (4)$$

At point (3), where the ring load acts, the jump correction term Δm accounts for the concentratedload of 3000 kg·cm/cm' applied along the circumference. At the boundary point (6), the simply supported conditions $m_6 = 0$ and $w_6 = 0$ are enforced with a fictitious node.

The resulting 14×14 system of linear algebraic equations is solved by matrix inversion, yielding the deflection vector: $W_0 = 0.5961$ cm, $W_1 = 0.5776$ cm, $W_2 = 0.5229$ cm, $W_3 = 0.4316$cm, $W_4 = 0.3032$ cm, $W_5 = 0.1556$ cm.

Using the computed deflections, bending moments in both radial and circumferential directions are calculated at each node using the finite difference expressions for M_R and M_θ. The maximum radial moment at point 1 is MR = 967.8 kg·cm/cm' (analytical: 937.5, error 3.13%), and t he maximum circumferential moment is $M_\theta = 2317.4$ kg·cm/cm' (analytical: 2062.5, error 10.99%). The radial moment shows excellent agreement with the analytical solution, while the circumferential moment, being more sensitive to the angular discretization, exhibits a larger but still acceptable error for engineering purposes

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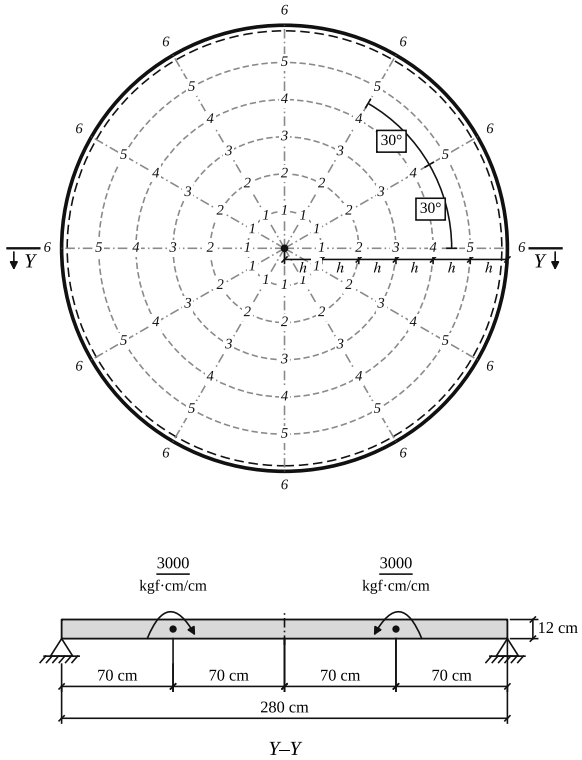


Fig. 2. Circular plate discretized with radial step h = a/6 = 23⅓ cm under ring load of 3000 kg·cm/cm' (cross-section Y-Y shown below)

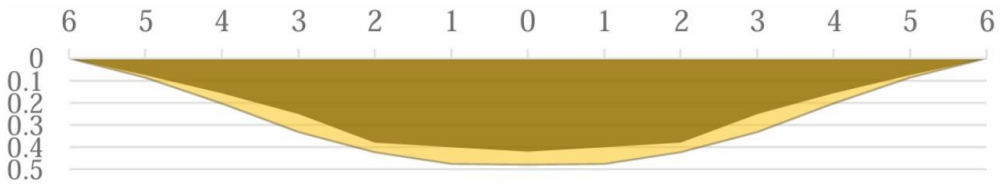


Fig. 3. Comparison of deflection profiles: analytical solution vs. GFDM results

Table 1. Comparison of maximum bending moments for the illustrative example

Method	M_R^{max} (kg·cm/cm')	M_θ^{max} (kg·cm/cm')
GFDM	967.8	2317.4
Analytical [15]	937.5	2062.5
Error (%)	3.13	10.99

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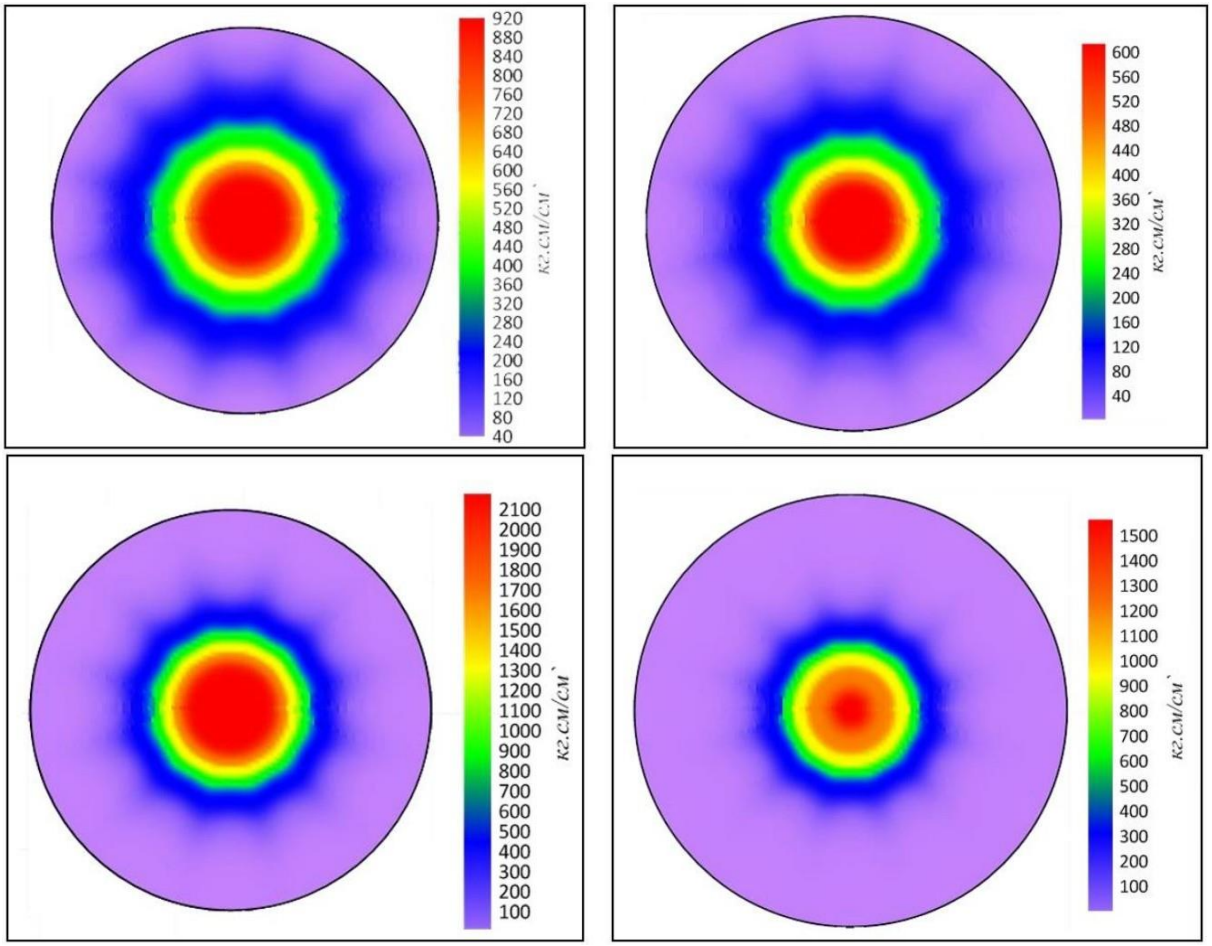


Fig. 4. Contour plots of bending moments: top row - radial M_R (left: GFDM, right: analytical); bottom row - circumferential M_θ (left: GFDM, right: analytical)

4 CONCLUSION

The generalized finite difference method of Gabbasov provides an effective numerical tool for the engineering analysis of thin isotropic circular plates in polar coordinates. Its principal advantages over the classical FDM are the ability to handle discontinuous distributed loads, concentrated forces, and ring loads without artificial mesh densification, and the resulting improvement in accuracy at moderate grid densities. For static problems, the GFDM achieves errors below 5% relative to analytical solutions for smooth loading cases even at coarse grids, and below 15% for discontinuous loads upon moderate refinement. The illustrative example demonstrates that the method yields maximum radial momenterrors of 3.13% and circumferential moment errors of about 11%, which are acceptable forengineering calculations with a relatively coarse grid of h = a/6. The FEM is preferred when complex boundary conditions or loads concentrated near the plateperiphery are dominant. The GFDM is recommended as an accurate and transparent alternative forstatic analysis of circular plates, particularly for educational and preliminary design purposes, where implementation in standard tools such as Mathcad and MS Excel provides accessibility without specialized software.

Future work should investigate the extension of the GFDM to plates with variable thickness, elastic foundation interaction, and dynamic loading conditions including free vibration and seismicresponse problems

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CONFLICT OF INTEREST

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INFORMATION ABOUT THE AUTHORS

Yerbolatova Dilnaz Yerbolatkyzy – Master's student, Department of Construction,International Educational Corporation (KazGASA), Almaty, Kazakhstan, eerbolatovad@gmail.com

Polyakova Irina Mikhailovna – Scientific Supervisor, Department of Construction,International Educational Corporation (KazGASA), Almaty, Kazakhstan, i.poliakova@kazgasa.kz

AUTHORS CONTRIBUTION

Yerbolatova D.Ye. – development of the numerical scheme; software implementation in Mathcad and MS Excel; conducting calculations; writing the article; preparation of tables andfigures; analysis of results.

Polyakova I.M. – scientific guidance; research concept; verification of methodology;scientific editing of the manuscript; final conclusions.

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